Multi-Shaker Random Mode Testing

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A mode survey test procedure that combines multi-shaker correlated broad-band random excitation of the test article with the mode isolation features of multiple shaker excitation is described. The procedure takes advantage of multiple shaker excitation to best isolate the target mode response from all other modes. Mode parameters are then established from direct observation of frequency response functions. The proposed test procedure derives from the recognition that response data of a structure excited at multiple locations by proportional randomly varying forces can be analyzed such that the results are equivalent to those obtained had the excitation been harmonic. The test procedure is described in detail and demonstrated by analytical simulation.

Nomenclature

$\{A\}$	= vector of elements A_ℓ
A_ℓ	= factor used to scale random excitation time history at ℓth degree of freedom
$\bar{F}$	= generalized force
$\{\hat{F}_k\}$	= vector of forces for excitation at ω_k
$\{f_n\}$	= vector of natural frequencies, Hz
$f(t)$	= random excitation time history
$f(\omega)$	= Fourier transform of $f(t)$
$[I]$	= identity matrix
i	= $\sqrt{-1}$
$[M]$	= mass matrix
$[\bar{M}]$	= generalized mass matrix
Ω'	= real component of admittance, Eq. (6)
$i\Omega''$	= imaginary component of admittance, Eq. (6)
ω	= frequency of excitation
ω_a, ω_b	= half-power point frequencies
ω_n	= circular natural frequency
ω_t	= target mode natural frequency
$\{q(t)\}$	= vector of modal displacements
$\{\dot{q}(t)\}$	= vector of modal velocities
$\{\ddot{q}(t)\}$	= vector of modal accelerations
$\{\ddot{q}(\omega)\}$	= Fourier transform of $\{\ddot{q}(t)\}$
$[\phi]$	= matrix of normal modes
$\phi_{\ell k}$	= kth mode shape value at degree of freedom ℓ
$\{\hat{\phi}_k\}$	= estimate of kth mode shape vector, elements of shaker position coordinates
$\{X(t)\}$	= vector of physical displacements
$\{\ddot{X}(\omega)\}$	= Fourier transform of $\{\ddot{X}(t)\}$
X_ℓ	= test article coordinate
ζ	= critical damping ratio

Subscripts

k	= counter of modal coordinates
ℓ	= counter of physical coordinates
m	= number of modes in isolation group
r	= number of physical coordinates
t	= target mode

Introduction

To characterize dynamic models of complex structures accurately, experimental measurement of the structure's natural modes of vibration is required. The measurement of these modes is typically a costly and technically demanding task that requires both experimental and analytical expertise. Over the years, methodologies have been developed to improve the accuracy of measured modes and to reduce the complexity, and therefore the cost, of the experimental procedures. Generally, these methodologies can be categorized into two groups. The first group consists of procedures that attempt to establish natural modes of vibration by direct measurement of the test article's forced vibration. The second group consists of those procedures that attempt to identify natural modes of vibration by post-test analysis of frequency response functions or post-test analysis of time domain free response data.

To date, procedures that attempt to establish natural modes of vibration by direct measurement use one or more shakers to exert sinusoidally varying forces on the test article. The frequency of excitation and the relative force levels of the multiple shakers are adjusted to best isolate the target mode response from all other modes. The mode parameters are then established from direct measurement of the forced vibration.

Over the years, test methodologies for adjusting the relative force levels of the multiple shakers have been proposed (e.g., Refs. 1-3). These procedures have relied on phase coherence of response (Ref. 4) as the criterion for successful isolation of a target mode. For example, the methodology introduced by Lewis and Wrisley[1] requires that the shakers be brought into play one at a time. As each new shaker is introduced, all the shaker force levels are readjusted in an attempt to obtain a force distribution in phase with response velocity, and thus establish displacement responses at all degrees of freedom in phase with each other. More recently, Anderson[5] proposed a test procedure that does not rely on phase coherence of response as a criterion for mode isolation. Instead, the procedure relies on maximizing a target mode quadrature response relative to the quadrature responses of modes close in frequency to the target mode. This is accomplished by using successive measured estimates of shaker location mode shape values to establish at target mode excitation vector nearly orthogonal to modes close in frequency to the target mode.

Several investigators have proposed test and analysis procedures that attempt to establish natural modes of vibration by post-test analysis of frequency response functions. Morosow and Ayre[6] and Stahle[7] have proposed analysis procedures to extract mode shapes from measured response to nonselective sinusoidal excitation. More recently, the introduction of the Fast Fourier Transform (FFT) algorithm and the availability of digital data processing equipment have led to the development of test procedures that make no attempt to cause the test article to vibrate at discrete frequencies. Instead, all modes within the frequency range of interest are simultaneously excited with either a single broadband randomly varying force, multiple uncorrelated

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broadband randomly varying force, or an impulsive force. The response data are then digitally processed into estimates of steady-state frequency response functions, from which modal parameters are estimated using any one of several candidate procedures (e.g., Refs. 8-10).

In addition to the frequency domain identification procedures, Ibrahim and Mikulick[11,12] have proposed analysis procedures that attempt to establish natural modes of vibration by post-test analysis of measured time domain free response data. Although several technical papers discussing these procedures have been published (e.g., Refs. 11, 12, and 13), test problems presented to date have emphasized the identification of natural frequencies and damping values. The procedures' ability to identify mode shapes of complex structures still needs to be demonstrated.

In the past few years, several mode survey tests on complex structures have been performed such that selected modes were measured using both multi-shaker sine-dwell and single-point-random test techniques. With mode orthogonality used as a comparison criterion, published evidence (Refs. 14, 15) and other unpublished test results indicate that for complex structures, multi-shaker sine-dwell test techniques presently yield better results than do single-point-random test and mode identification techniques. An advantage presently enjoyed by the multi-shaker sine-dwell test technique is that the modes of vibration are established before disassembly of the test setup, and any deficiencies in the quality of the data (as caused by a malfunctioning accelerometer, for example) can be reconciled. With the post-test mode identification procedures, it is more difficult to identify "bad" data because very often the modes of vibration are not available until months after the test, and no additional data can be collected because schedule considerations have forced the disassembly of the test setup. Another advantage enjoyed by the multi-shaker sine-dwell test technique is that the modal parameters are the result of direct observations of the test article's forced vibration. With the random or impulsive test and mode identification techniques, the modal parameters are established from "measured" frequency response functions as "best estimates" and, therefore, are dependent on the numerical techniques (and their inherent assumptions and limitations) used in the identification process.

As discussed above, mode survey test procedures can be catagorized according to whether they attempt to establish natural modes of vibration by direct measurement or attempt to identify modes of vibration by post-test analysis of response to nonselective excitation. It was also noted that procedures establishing natural modes of vibration by direct measurement use one or more shakers to exert sinusoidally varying forces on the test article. The frequency of excitation and the relative force levels of the shakers are adjusted to best isolate the target mode response. Mode parameters are then established from direct measurement of the forced vibration.

The purpose of this paper is to introduce a mode survey test procedure that uses correlated broadband randomly varying forces to accomplish the same task that until now required harmonic excitation of the test article. The procedure derives from the recognition that response data of a structure excited at multiple locations by proportional randomly varying forces can be analyzed such that the results are equivalent to those obtained had the excitation been harmonic. The procedure takes advantage of multiple shaker excitation to best isolate the target mode response from all other modes. Mode parameters can then be established from direct observation of frequency response functions.

Frequency Response Functions – A Review

The matrix differential equation of motion that describes the behavior of a structure subjected to random excitation $\{A\}f(t)$ is written in modal coordinates as

$$[I]\{\ddot{q}(t)\}+[2\zeta\omega_n]\{\dot{q}(t)\}+[\omega_n^2]\{q(t)\}=[\phi]^T\{A\}f(t) \tag{1}$$

where the coordinate transformation between physical coordinates $\{X(t)\}$ and modal coordinates $\{q(t)\}$ is defined by

$$\{X(t)\}=[\phi]\{q(t)\} \tag{2}$$

and the matrix of mode shape vectors $[\phi]$ has been normalized with respect to the system mass matrix $[M]$ such that

$$[\phi]^T[M][\phi]=[I] \tag{3}$$

For clarity of presentation, we will use the continuous infinite-range Fourier transform to establish frequency response functions. Note, however, that we will assume later that for our purposes, frequency response functions obtained with digital FFT technology are equivalent to frequency response functions obtained with the continuous infinite-range Fourier transform. Proceeding, we take the Fourier transform of Eq. (1) and solve for the Fourier transform of the modal coordinate acceleration vector to obtain

$$\{\ddot{q}(\omega)\}=-\omega^2([\omega_n^2-\omega^2]^2+[2\zeta\omega_n\omega]^2)^{-1}([\omega_n^2-\omega^2] - i[2\zeta\omega_n\omega])[\phi]^T\{A\}f(\omega) \tag{4}$$

where the kth coordinate equation has the following form:

$$\ddot{q}_k(\omega)=\left\{\frac{-\lambda_k^2[1-\lambda_k^2]+i2\zeta_k\lambda_k^3}{[1-\lambda_k^2]^2+[2\zeta_k\lambda_k]^2}\right\}\bar{F}_k f(\omega) \tag{5}$$

$$=\{\Omega_k'+i\Omega_k''\}\bar{F}_k f(\omega) \tag{6}$$

and

$$\lambda_k=\omega/\omega_{n_k} \tag{7}$$

$$\bar{F}_k=\sum_{\ell=1}^{r}\phi_{\ell k}A_\ell \tag{8}$$

A_ℓ = the factor used to scale the random excitation time history at degree of freedom ℓ, and $f(\omega)$ = the Fourier transform of the random excitation time history $f(t)$. Taking the Fourier transform of the second time derivative of Eq. (2) and using Eq. (6) to substitute for $\ddot{q}_k(\omega)$, we obtain for a typical physical coordinate acceleration

$$\ddot{X}_\ell(\omega)/f(\omega)=\sum_{k=1}^{r}\phi_{\ell k}\{\Omega_k'+i\Omega_k''\}\bar{F}_k \tag{9}$$

The right side of Eq. (9) is recognized to be the response function of the ℓth degree of freedom of a multi-degree-of-freedom structure that is being driven at multiple locations by a sinusoidally varying forcing function of frequency ω. The real part corresponds to the coincident response (i.e., the component of acceleration response that is colinear with the reference force time history) and the imaginary part corresponds to the quadrature response (i.e., the component of acceleration response that is 90 deg out of phase with the reference force time history). Therefore, Eq. (9) implies that the response data of a structure excited at multiple locations by proportional randomly varying forces can be analyzed such that the results are equivalent to those obtained if the structure were to be excited at those same locations by sinusoidally varying forces.

Mode Isolation Logic

Assume for the moment that the test article can be simultaneously driven at all its degrees of freedom by proportional randomly varying forces. Equation (9) then indicates that the test article can be made to vibrate in a single normal mode by appropriately adjusting the relative shaker force levels A_ℓ such that the generalized forces $\bar{F}_k$ are zero for all modes except the target mode. In practice, however, the number of degrees of freedom needed to represent a complex structure greatly exceeds the number of available shakers. Therefore, perfect isolation is not possible.

Fortunately, for practical purposes, some small amount of contamination in measured modes is tolerable. It has become a widely accepted practice to judge the quality of measured modes by their mutual orthogonality. This is accomplished by calculating the unit normalized generalized mass matrix and comparing the magnitudes of the off-diagonal terms to a predetermined value. It is generally accepted in the industry that modes of acceptable quality have been measured if these off-diagonal terms are no greater than 0.10. To a first approximation, this implies that a target mode is sufficiently isolated for measurement if the amplitude (the quadrature component of the total response) of the contaminating mode is no greater than 0.10 times the target mode amplitude, i.e.,

$$\Omega_k'' \bar{F}_k < (0.10)\, \Omega_t'' \bar{F}_t \tag{10}$$

Therefore, we shall proceed with the understanding that perfect isolation of a target mode is not necessary and that the number of shakers need not equal the number of degrees of freedom in the test article.

It was noted earlier that the response data of a structure excited at multiple locations by proportional randomly varying forces can be analyzed such that the results are equivalent to those obtained if the structure were to be excited by multiple sinusoidally varying forces. Therefore, it should be possible to adopt the basic principles of multi-shaker sine-dwell mode survey testing and apply them to multi-shaker random mode survey testing.

For the multi-shaker random mode survey test procedure proposed herein, the Anderson mode isolation logic of selective orthogonal excitation (SOREX)[5] has been adopted. The SOREX logic takes advantage of the natural selectivity of lightly damped structures to greatly amplify their forced responses at resonant frequencies. A consequence of this observation is recognition that careful attention to the selectivity of the applied forces generally is necessary only for modes in close frequency proximity to the target mode. This can best be illustrated by rewriting Eq. (10) as

$$(\Omega_k''/\Omega_t'')\,(\bar{F}_k/\bar{F}_t) < 0.10 \tag{11}$$

The above equation states that adequate target mode isolation from an off-resonance mode will exist if the product of the quadrature admittance ratio and generalized force ratio is less than 0.10.

Equation (11) is the essence of the SOREX procedure. Basically, the procedure recognizes that for lightly damped structures the ratio (Ω_k''/Ω_t'') will decrease rapidly with increasing frequency separation between the two modes. Thus, careful attention to the applied forces, which determine the ratio $(\bar{F}_k/\bar{F}_t)$, is necessary only for modes in close frequency proximity to the target mode. Data presented in Ref. 5 and test experience with complex spacecraft structures indicate that if a mode is removed in frequency from the target mode by more than about 10 to 15%, it can generally be ignored during measurement of the target mode. Therefore, the applied forces will generally only be required to isolate the target mode response from modes "close" in frequency.

To isolate target mode responses from modes close in frequency, the SOREX test logic is proposed. Basically, the procedure consists of using successive estimates of mode shape values to establish force vectors whose corresponding generalized force for the target mode is large in relation to the generalized forces of modes close in frequency. This is accomplished by first establishing isolation groups that consist of one or more target modes and all modes within about 10 to 15% in frequency. To obtain optimum isolation, the number of excitation shakers must equal the number of modes in the isolation group (it is possible to use the SOREX test logic when the number of modes in an isolating group exceeds the available shakers[5]; however, the isolation will generally not be as complete). Typically, for complex structures, each isolating group will consist of 2 to 12 modes. Therefore, a relatively small number of shakers will be required.

For each isolation group, the initial shaker force levels are established from frequency response functions obtained with single shaker excitation. The response functions are used to estimate the shaker location mode shape values, which are then used to calculate the relative force levels to be applied at those locations:

$$[\hat{F}_1, \hat{F}_2, \ldots \hat{F}_m] = [\bar{I}]\,([\hat{\phi}_1 \hat{\phi}_2 \ldots \hat{\phi}_m]^T)^{-1} \tag{12}$$

where

$\{\hat{F}_k\}$ = vector of forces for excitation of mode at ω_k,
$\{\hat{\phi}_k\}$ = estimate of kth mode shape vector (quadrature response), elements of shaker position coordinates only, and
$[\bar{I}]$ = matrix whose kth column represents ideal generalized force vector for isolating kth mode from other modes in the isolation group, typically an identity matrix. Each calculated force vector is then applied to the structure, the frequency of excitation adjusted to maximize quadrature response, and new values of $\{\hat{\phi}_k\}$ measured. These new mode shape values are then used in Eq. (12) to calculate a set of refined shaker force levels. If the number of shakers is equal to the number of modes in an isolation group, which we shall assume, convergence of the force vectors to those required for adequate mode isolation typically occurs within one or two iterations. Once isolation is achieved, the entire shape is measured and its orthogonality checked against the other measured mode shapes.

The test procedure proposed by Anderson was formulated for use with harmonic excitation of the test article. However, Eq. (9) indicates that if we excite the structure with multiple randomly varying forces, the response data can be analyzed such that the results are equivalent to those obtained had the excitation been harmonic. Therefore, it is possible to use the SOREX logic to establish the relative magnitudes of multiple randomly varying forces such that a target mode is isolated for measurement from modes close in frequency. We now have all the ingredients needed to propose a multi-shaker random mode survey test procedure.

Multi-Shaker Random Mode Survey Test Procedure

Combining multi-shaker correlated broadband random excitation with the basic principles of multi-shaker sine-dwell mode survey testing, the following procedure is established:

1) Use single-point random excitation to establish frequency response functions at all potential shaker positions and any coordinates whose response will aid in identifying all modes in the frequency range of interest.

2) Determine preliminary natural frequencies and predominant motions of each mode.

3) Measure all modes which are removed in frequency from all other modes (i.e., $0.85\,\omega_k < \omega_t < 1.15\,\omega_k$) by applying a randomly varying force at a location on the test article that exhibits strong response in the mode to be measured, and then processing the response data into frequency response func-

tions. Steps 11 and 12 define how the modal parameters are to be extracted from the response functions.

4) Establish isolation groups of modes sufficiently close in frequency to warrant inclusion in determining multiple excitation force levels (i.e., $0.85\,\omega_t < \omega_k < 1.15\,\omega_t$).

5) For each isolation group, select for shaker locations those coordinates with strong resonances and phase reversals between modes.

6) For each of the selected coordinates, extract from the imaginary part of the frequency response functions established in Step 1 the peak values associated with each of the modes in an isolation group and form $[\hat{\phi}]$.

7) Using Eq. (12), calculate $[\hat{F}_1, \hat{F}_2, \ldots \hat{F}_m]$.

8) Apply random excitation at the coordinates selected in Step 5 such that the relative force levels of the shakers are defined for each target mode by $\{\hat{F}_t\}$.

9 For each $\{\hat{F}_t\}$, select the force signal from one shaker as a reference signal and process the response data from each shaker location into frequency response functions.

10) Review frequency response functions from Step 9 to determine if adequate isolation has been achieved. If isolation has not occurred, form a new $[\hat{\phi}]$ with updated shaker location mode shape values from Step 9 and repeat Steps 7 through 10.

11) Record the mode shape as the maximum values of the imaginary part of the frequency response function of each

Table 1 Summary of numerical simulation iterations

	Initial excitation			First iteration			Second iteration		
	ϕ_1	ϕ_2	ϕ_3	ϕ_1	ϕ_2	ϕ_3	ϕ_1	ϕ_2	ϕ_3
A_1	0.00	0.00	0.00	0.17	−0.35	1.02	0.26	−0.11	−7.11
A_3	0.00	0.00	0.00	0.52	−0.17	−0.83	0.34	−0.12	6.95
A_4	1.00	1.00	1.00	5.94	9.37	−2.01	17.32	4.27	−11.23
$\bar{F}_1$	−0.11	−0.11	−0.11	−2.09	0.00	0.00	−3.15	0.00	0.15
$\bar{F}_2$	−0.03	−0.03	−0.03	0.43	−0.68	0.05	0.01	−0.32	1.54
$\bar{F}_3$	−0.01	−0.01	−0.01	0.67	0.43	−4.15	0.01	0.00	31.45
$\bar{F}_4$	1.06	1.06	1.06	6.18	10.08	−2.19	18.30	4.59	−11.88
$\bar{F}_5$	−0.07	−0.07	−0.07	−0.31	−0.68	0.16	−1.06	−0.31	0.70
$\bar{F}_6$	2.98	2.98	2.98	17.69	27.88	−5.99	51.53	12.70	−33.41
$[\bar{M}]$	1.00	0.21	0.33	1.00	−0.01	0.00	1.00	0.00	0.00
	0.21	1.00	0.46	−0.01	1.00	0.05	0.00	1.00	0.00
	0.33	0.46	1.00	0.00	0.05	1.00	0.00	0.00	1.00

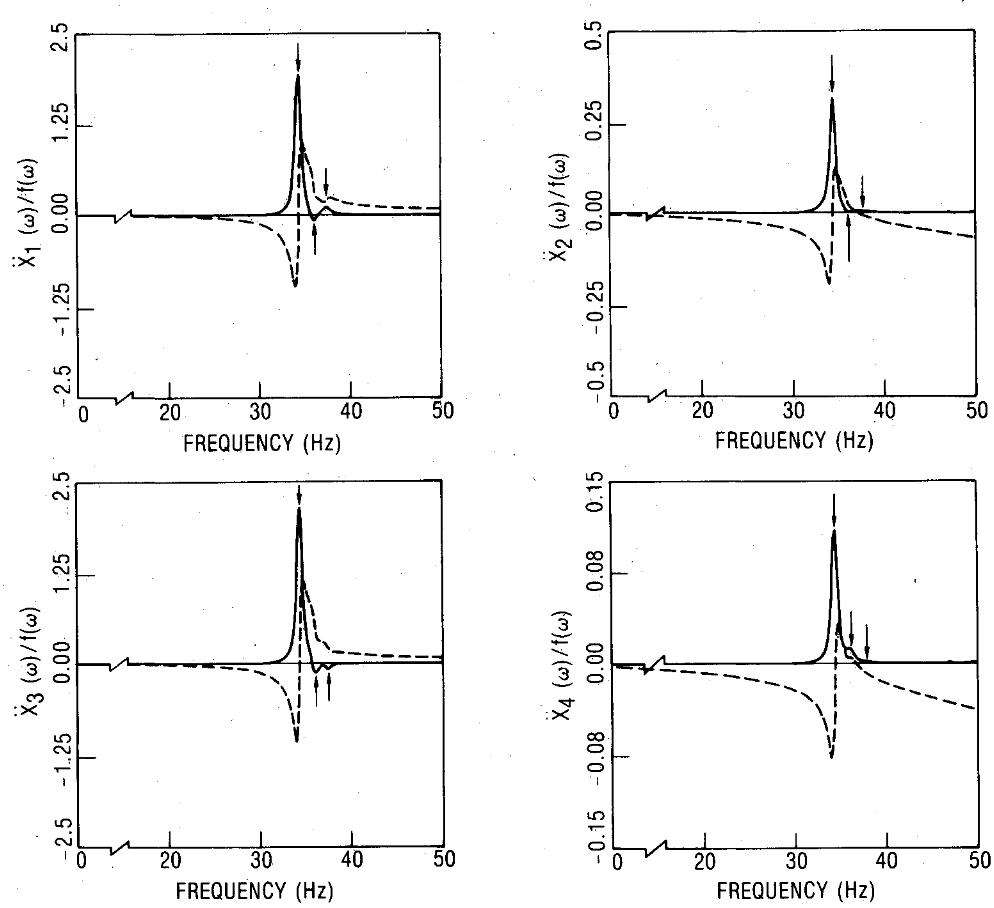

Fig. 1 Frequency response functions: random excitation at Coordinate X_4, real component---, imaginary component___.

coordinate. The natural frequency is recorded as the frequency at which the imaginary part is maximum.

12) Use the real part of the frequency response function to calculate damping:

$$\zeta_t = \omega_a^2 - \omega_b^2 / 4\omega_t^2 \tag{13}$$

where ζ_t = critical damping ratio, ω_a, ω_b = frequencies, in the vicinity of the target mode frequency, at which the real part of the frequency response function is maximum or minimum, and ω_t = target mode natural frequency.

13) Repeat Steps 5 thorugh 12 for each isolation group.

14) Check orthogonality of all measured modes. If unacceptable contamination exists, expand the isolation group of the contaminated mode to include the contaminating modes, establish a refined force vector, and remeasure the contaminated mode. Note that with the measured mode set, a good approximation to the generalized forces can be calculated. These forces can be used as an aid in detemining which mode in a pair of modes exhibiting poor orthogonality should be remeasured.

Demonstration of Procedure by Numerical Simulation

To demonstrate the multi-shaker random mode survey test procedure, an analytical simulation of a test was performed and the results compared to a) those obtained using the multi-shaker sine-dwell mode survey test procedure presented in Ref. 5, and b) the eigenvectors obtained by solving the system undamped eigenvalue problem. The analytical model employed in the simulation had the following dynamic properties:

$$[\phi] = \begin{bmatrix} 1.954 & -0.742 & -2.360 & -0.209 & 0.127 & 0.009 \\ 0.318 & -0.060 & -0.020 & 1.959 & -2.340 & -0.763 \\ 2.156 & -0.933 & 2.103 & -0.205 & 0.124 & 0.008 \\ 0.110 & 0.029 & -0.005 & 1.064 & -0.066 & 2.975 \\ 0.174 & 0.189 & -0.001 & 2.211 & 2.116 & -0.752 \\ 1.179 & 2.922 & 0.072 & -0.232 & -0.113 & 0.008 \end{bmatrix} \quad \{f_n\} = \begin{Bmatrix} 34.462 \\ 36.098 \\ 37.503 \\ 121.179 \\ 166.086 \\ 356.219 \end{Bmatrix} \quad \{\zeta\} = \begin{Bmatrix} 0.01 \\ 0.01 \\ 0.01 \\ 0.02 \\ 0.02 \\ 0.03 \end{Bmatrix}$$

The response of the model to multiple sinusoidally varying forces was obtained by establishing the closed-form complex solution to the equations of motion; the real part of the solution being the coincident response and the imaginary part of the solution being the quadrature response. The response to multiple randomly varying forces was established by numerically integrating the equations of motion. The computed response time histories and the reference force time history were then digitally processed into frequency response functions from which the modal parameters were directly inferred (without curve fits).

The test simulation was initiated by exciting with a randomly varying force the analytical test article at coordinate X_4 and processing the response data at selected coordinates into frequency response functions. From these frequency response functions, shown in Fig. 1 for the frequency range 20-50 Hz, it was determined that three modes were present in the frequency range of interest, 0-50 Hz. (For ease of comparison, all frequency response functions have been normalized to provide unit generalized mass for each target mode.) Furthermore, it was concluded that all three modes were sufficiently close in frequency to warrant inclusion in the same isolation group.

For the multi-shaker excitation phase of the simulation, coordinates X_1, X_3, and X_4 were selected as shaker locations. Estimates of mode shape values at these coordinates were obtained from the frequency response functions established in the single-shaker random excitation phase of the simulation. The mode shape values were taken as the peak imaginary response at the estimated natural frequency of each mode. If an obvious peak did not exist, as was the case of the third mode at coordinate X_4 (see Fig. 1), the value at the estimated frequency of the mode was used. The estimates of mode shape values were then used in Eq. (10) to establish the relative excitation force amplitudes needed to improve the isolation of each target mode.

The acceleration response of coordinate X_1 at each iteration is shown in Fig. 2. As the figure illustrates, the isolation of each target mode improves as the relative excitation force levels become more refined. This is especially evident for the second mode, where it can be observed that for single shaker excitation the dominant response of coordinate X_1 was that of mode 1. The first iteration with three shakers succeeded in isolating the second mode from the first mode; however, adequate isolation from both the first and third modes required one more iteration.

The calculated shaker force levels, the resulting generalized forces, and the unit normalized generalized mass matrix obtained at each iteration are presented in Table 1. As the table illustrates, the single shaker excitation resulted in a relatively large generalized force for the first mode, which partially explains the dominant response of this mode observed at coordinates X_1, X_2, X_3, and X_4 (see Fig. 1). Subsequently, the application of multiple shakers, as Table 1 also illustrates, resulted in the target mode generalized force being maximized relative to the generalized forces of the other modes in the isolation group. In addition, the generalized mass matrices indicate that the test success criterion of all off-diagonal terms in the unit normalized generalized mass matrix being no greater than 0.10 was satisfied after the first iteration.

The mode shapes obtained at each iteration are compared in Table 2 to the mode shapes obtained using the multi-shaker sine-dwell mode survey test procedure presented in Ref. 5, and to the eigenvectors obtained by solving the system undamped eigenvalue problem. For the analytical model employed in the simulation, mode shapes obtained with the multi-shaker random excitation procedure are indistinguishable from mode shapes established with the multi-shaker sine-dwell excitation procedure advocated in Ref. 5 (see Table 2). In addition, excellent agreement exists between the exact mode vectors and those established in the simulations.

Some Practical Considerations

The actual performance of a mode survey test is more difficult than the mathematical simulations lead us to believe. Test experience indicates that interaction between the test article and the excitation shakers introduces phase distortions between the shaker-applied forces. For harmonic excitation, this problem is overcome by delaying in time one shaker force signal relative to another and thus adjusting for any phase

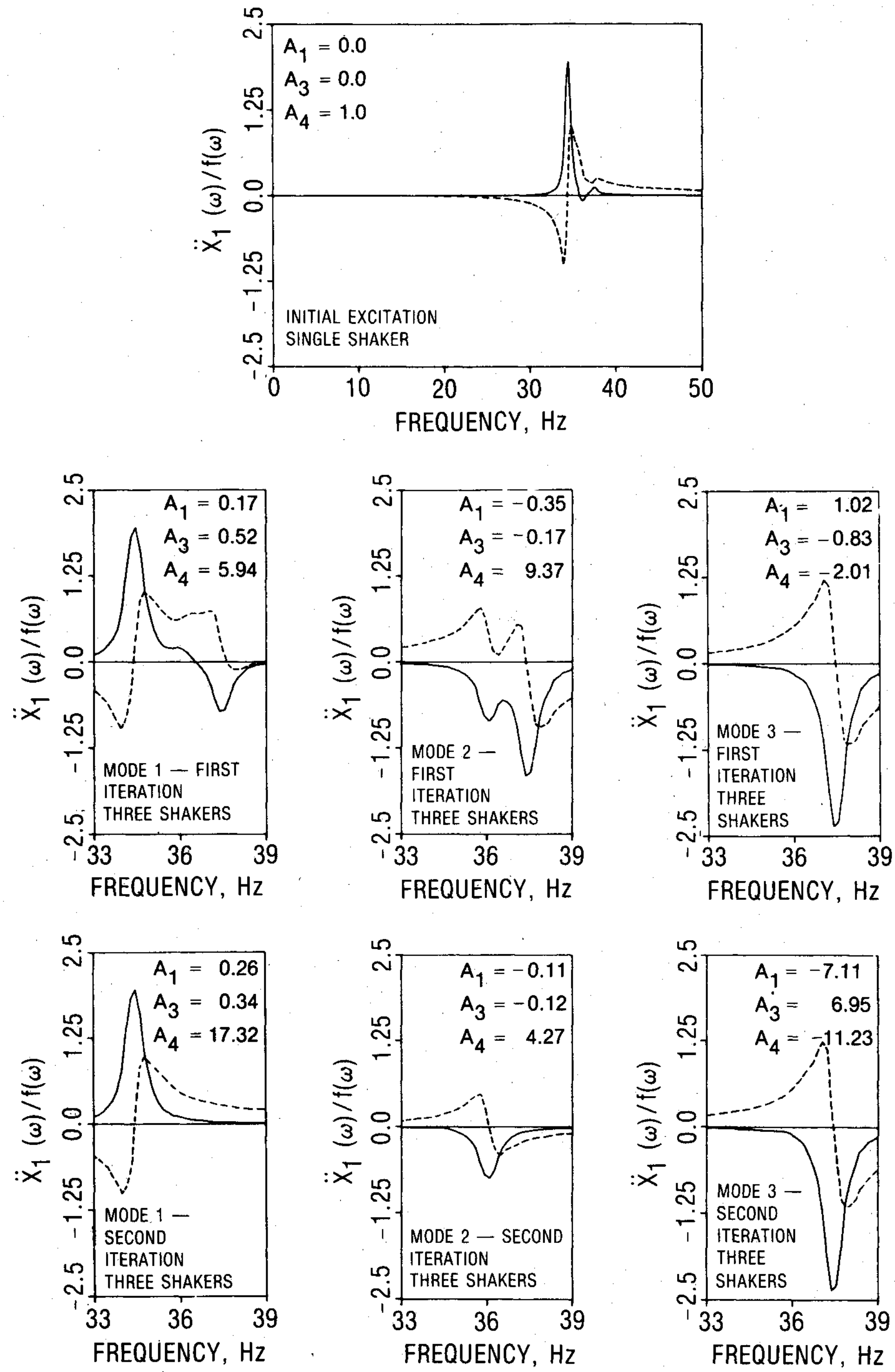

Fig. 2 Response of Coordinate X_1, real component---, imaginary component___.

distortions. This task is considerably more difficult for random excitation, because the phase distortions will vary not only from shaker to shaker but also will vary as a function of frequency at each shaker. At least conceptually, this problem can be minimized with a closed-loop control system. This added complexity needs to be considered when contemplating a mutli-shaker random mode survey test.

As discussed previously, target mode isolation from modes that are not included in the isolation group is due primarily to separation in mode frequencies. However, the generalized force of a mode not included in the isolation group occasionally is sufficiently large such that 10 to 15% frequency separation from the target mode is not sufficient. Two corrective actions can be taken. The simpler approach is to include the contaminating mode in the isolation group, deploy an additional shaker, and remeasure the contaminated mode. The second approach is to move the shaker that is primarily responsible for the large response of the contaminating mode. First, the measured target mode and measured contaminating mode must be normalized to unit generalized mass. Next, the mode shape values at the shaker locations should be compared. Usually, at one of the shaker locations the contaminating mode will have a considerably larger response than the target mode. The shaker attached at that location should be redeployed, preferably to a location where the target mode response is greater than that of the contaminating mode.

Although numerical simulations can serve as valuable research tools, the true test of a mode survey test procedure occurs in the laboratory. In early 1982, the proposed multi-shaker random mode survey test procedure was successfully used to establish the modes of vibration of a spacecraft structure. During the test, several modes were isolated for measurement using multi-shaker (three shakers) correlated

Table 2 Comparison of mode shapes: multi-shaker random, multi-shaker sine-dwell, and exact

	Sine-dwell			Random			Exact
	Iteration			Iteration			
	Initial	First	Second	Initial	First	Second	
ϕ_1	1.948	1.950	1.955	1.947	1.951	1.954	1.954
	0.318	0.319	0.318	0.316	0.318	0.317	0.318
	2.145	2.171	2.157	2.143	2.174	2.156	2.156
	0.111	0.110	0.111	0.110	0.110	0.110	0.110
	0.176	0.173	0.174	0.175	0.172	0.173	0.174
	1.211	1.157	1.179	1.215	1.150	1.178	1.178
ϕ_2	−0.374	−0.827	−0.741	−0.309	−0.856	−0.741	−0.742
	−0.003	−0.061	−0.060	0.005	−0.061	−0.060	−0.060
	−0.574	−0.858	−0.932	−0.512	−0.829	−0.932	−0.933
	0.049	0.029	0.030	0.053	0.029	0.029	0.029
	0.218	0.189	0.190	0.220	0.189	0.189	0.189
	3.079	2.922	2.923	3.096	2.922	2.923	2.922
ϕ_3	2.398	−2.361	−2.358	2.381	−2.361	−2.358	−2.360
	0.106	−0.020	−0.020	0.117	−0.020	−0.020	−0.020
	−1.506	2.103	2.105	−1.482	2.103	2.106	2.103
	0.058	−0.005	−0.005	0.062	−0.005	−0.005	−0.005
	0.137	−0.001	−0.001	0.152	−0.001	−0.001	−0.001
	1.395	0.074	0.065	1.451	0.075	0.062	0.072

broadband random excitation. Each shaker was commanded from the same signal generator, however, no closed-loop control system was used. Of the tend modes measured, eight satisfied the test orthogonality success criterion of all off-diagonal terms in the unit normalized generalized mass matrix being no greater than 0.10. The one off-diagonal term greater than 0.10, which was 0.19, occurred between a pair of modes in which the resonant components were inaccessible to the shakers.

Summary

A mode survey test procedure has been introduced that combines random excitation and digital processing of response data with the mode isolation features of mutliple shaker excitation. The proposed procedure uses correlated broadband randomly varying forces to accomplish the same task that up to now required harmonic excitation of the test article. The procedure derives from the recognition that response data of a structure, excited at multiple locations by proportional randomly varying forces, can be analyzed such that the results are equivalent to those obtained had the excitation been harmonic. The procedure takes advantage of mutliple shaker excitation to best isolate the target mode response from all other modes. Mode parameters are then established from direct observations of frequency response functions.

The multi-shaker random mode survey test procedure has been demonstrated herein by analytical simulation of a test. The results of the simulation are compared, with excellent agreement obtained, to values established with a multi-shaker sine-dwell test procedure and to the exact results.

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